

BUDDHA SERIES

(Unit Wise Solved Question & Answers)

Programme – B.Sc. Maths 3rd year 5th Semester

College – Buddha Degree College

(DDU Code-859)

Department: Science

Course code: PHY 302

Course title: Quantum Mechanics and Spectroscopy

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Unit-1

1. Which of the following is a scalar operator in quantum mechanics?

A. Momentum operator \widehat{p} B. Angular momentum operator \widehat{L} C. Identity operator \widehat{I} D. Position operator \widehat{x} Answer: C

2. An operator \hat{A} acting on a vector space must satisfy:

A. Linearity B. Hermiticity C. Unitarity D. Normality Answer: A

3. The matrix representation of an operator depends on:

A. The choice of basis B. The energy eigenvalues C. Whether the operator is Hermitian D. Commutation relations **Answer:** A

4. A projector operator satisfies:

A. $\widehat{P}^2=\widehat{I}$ B. $\widehat{P}^2=0$ C. $\widehat{P}^2=\widehat{P}$ D. $\widehat{P}^\dagger=-\widehat{P}$ Answer: C

5. The commutator $[\widehat{x},\widehat{p}]$ equals:

A. 0 B. $i\hbar$ C. $-i\hbar$ D. $i\hbar \hat{I}$ Answer: D (same as B, but operator form)

6. Which is true: $[\widehat{L}_x, \widehat{L}_y] =$

A. 0 B. $i\hbar \hat{L}_z$ C. $-i\hbar \hat{L}_z$ D. $i\hbar (\hat{L}_x + \hat{L}_y)$ Answer: B

7. Commutator algebra follows:

A. Bilinearity B. Jacobi identity C. Antisymmetry D. All of the above **Answer:** D

8.
$$[\hat{H}, t] =$$

A. $i\hbar$ B. 0 C. $-i\hbar \hat{I}$ D. Undefined **Answer:** B (since time is a parameter, not operator)

9. A non-degenerate eigenvalue has:

A. One eigenvector B. Two eigenvectors C. No eigenvectors D. Infinite eigenvectors ${\bf Answer:}\ {\bf A}$

10. Two eigenstates with same eigenvalue are:

A. Orthogonal B. Degenerate C. Non-orthogonal D. Unique $\bf Answer:~B$

11. Eigenvalue equation: $\widehat{A}\psi = a\psi$. Here a is:

A. Operator B. Eigenfunction C. Eigenvalue D. Expectation value Answer: C

12. Expectation value: $\langle \widehat{A} \rangle =$

A. $\int \psi^* A \psi \, dx$ B. $\psi^* \widehat{A} \psi$ C. $\int \psi^* \widehat{A} \psi \, dx$ D. $\widehat{A} \psi$ Answer: C

13. Which of these is true for a Hermitian operator \widehat{A} ?

A. $\widehat{A}^\dagger=-\widehat{A}$ B. $\widehat{A}^\dagger=\widehat{A}$ C. $\widehat{A}^2=\widehat{I}$ D. $\widehat{A}^*=\widehat{A}$ Answer: B

14. Hermitian operators have:

A. Complex eigenvalues B. Pure imaginary eigenvalues C. Real eigenvalues D. No eigenvalues ${\bf Answer:}\ {\bf C}$

15. The momentum operator in one dimension:

A. $-i\hbar \frac{d}{dx}$ B. $i\hbar \frac{d}{dx}$ C. $-\hbar^2 \frac{d^2}{dx^2}$ D. $i\hbar x$ **Answer:** A

16. The raising operator \hat{L}_{+} is defined as:

A.
$$\widehat{L}_x-i\widehat{L}_y$$
 B. $\widehat{L}_x+i\widehat{L}_y$ C. $\widehat{L}_y+i\widehat{L}_z$ D. $\widehat{L}_x+\widehat{L}_z$ Answer: B

17. The commutator $[\hat{p}_x, \hat{p}_y] =$

A.
$$i\hbar$$
 B. $-i\hbar$ C. 0 D. Depends on coordinates **Answer:** C

18. If $\widehat{A}|a\rangle=a|a\rangle$ and $\widehat{B}|a\rangle=b|a\rangle$, with [A,B]=0, then $|\mathbf{a}\rangle$ is:

A. Degenerate B. Simultaneous eigenstate C. Superposition D. Orthogonal
$${\bf Answer:}\ {\bf B}$$

19. Expectation value interpretation:

A. Most probable value B. Fixed measurement result C. Average over many measurements D. Variance of operator **Answer:** C

20. Which operator is guaranteed Hermitian in quantum mechanics?

A.
$$i\hbar \frac{d}{dx}$$
 B. $-\hbar^2 \frac{d^2}{dx^2}$ C. Position operator \hat{x} D. Time evolution operator $e^{-iHt/\hbar}$ Answer: C

21. Proof of Hermitian nature requires:

A. Integration by parts B. Real wavefunctions C. Non-normalizable wavefunctions D. Ignoring boundary terms **Answer:** A

22. A Hermitian operator's expectation value is:

A. Always imaginary B. Always negative C. Always real D. Unpredictable ${\bf Answer:}~{\bf C}$

23. The commutator $[\widehat{H},\widehat{p}]$ determines:

A. Energy levels B. $[\widehat{x}(t),$ dynamics] Time evolution of momentum via Heisenberg eqn C. Conservation of mass D. Degeneracy **Answer:** B

24. Linear superposition of eigenfunctions also satisfies:

A. Momentum eigenvalue for each B. Schrödinger equation only if degenerate C. Time-independence D. Orthogonality automatically **Answer:** B

25. In position basis, \hat{x} is:

A. Multiplication by x B. Derivative w.r.t x C. Fourier transform D. Second derivative **Answer:** A

26. The operator $\widehat{L}^2 = \widehat{L}_x^2 + \widehat{L}_y^2 + \widehat{L}_z^2$ commutes with:

A. Each of $\widehat{L}_x,\widehat{L}_y,\widehat{L}_z$ B. None of them C. Only \widehat{L}_z D. Momentum **Answer:** A

27. The time–energy commutator $[\widehat{H},\widehat{t}]$:

A. Is zero because t isn't an operator B. Equals $i\hbar$ C. Equals $-i\hbar$ D. Defines time operator analogously to position **Answer:** A

28. Degenerate subspace:

A. Contains only one eigenvector B. Contains eigenvectors of distinct eigenvalues C. Contains multiple eigenvectors with same eigenvalue D. Is always two-dimensional **Answer:** C

29. For Hermitian \widehat{A} , eigenstates corresponding to different eigenvalues are:

A. Always orthogonal B. Never orthogonal C. May be orthogonal if degenerate D. Always degenerate **Answer:** A

30. Which is NOT a property of Hermitian operators?

A. Real eigenvalues B. Orthogonal eigenstates C. Generate unitary evolution D. Expectation value real **Answer:** C (Unitary evolution is generated by $i\hat{H}$, not \hat{H} alone)

Unit-2

 The Heisenberg uncertainty principle stems fundamentally from which property? A) Time dependence of operators B) Non-commutativity of Hermitian operators C) Orthogonality of eigenstates D) Normalization of wavefunctions

Answer: B. Non-commutativity underlies the general uncertainty relation

The Robertson uncertainty relation is: A) $\triangle A \triangle B \le |\langle [A,B] \rangle| \ B) \triangle A \triangle B \ge \frac{1}{2} |\langle [A,B] \rangle| \ C) \triangle A \triangle B \ge |\langle \{A,B\} \rangle| \ D) \triangle A \triangle B \le \frac{1}{2} |\langle \{A,B\} \rangle|$

Answer: B

The more general Schrödinger form includes which component? A)
 Commutator squared B) Anticommutator term C) Determinant of operators
 D) Energy eigenvalues

Answer: B

3. Kennard's form for position & momentum is: A) $\Delta x \Delta p \ge \hbar$ B) $\Delta x \Delta p \ge \hbar/2$ C) $\Delta x \Delta p \ge 2\hbar$ D) $\Delta x \Delta p \ge 2\hbar^2$

Answer: B

Which derivation tool is used for the general uncertainty principle? A)
 Integration by parts B) Cauchy–Schwarz (Schwarz) inequality C) Taylor series
 D) Fourier transform

Answer: B

5. For non-commuting operators A and B, the commutator [A,B] equals: A) AB + BA B) AB – BA C) BA – AB D) 0

Answer: B

6. **Which conjugate pair demonstrates the minimum uncertainty in ground state?** A) Energy–time B) Position–momentum C) Angular momentum–angle D) Spin–magnetic field

Answer: B.

7. Harmonic oscillator ground state saturates uncertainty because Δx Δp
 =: A) ħ B) ħ(n + 1/2) C) ħ/2 D) ħ(n)

Answer:

9. Time-dependent Schrödinger equation is: A) $\hat{H}\psi$ = E ψ B) $i\hbar\partial\psi/\partial t$ = $\hat{H}\psi$ C) - $\hbar^2\nabla^2\psi$ = E ψ D) $\partial\psi/\partial x$ = $\hat{H}\psi$

Answer: B.

10. Time-independent Schrödinger equation for energy eigenstates: A) $\hat{H}\psi = i\hbar\partial\psi/\partial t$ B) $\hat{H}\psi = E\psi$ C) $\hat{H}\psi = \nabla^2\psi$ D) $\hat{H}\psi = 0$

Answer: B.

11.**In Schrödinger representation, continuity equation** ∂ρ/∂t + ∇·**J** = 0 **expresses:** A) Conservation of energy B) Conservation of probability C) Conservation of momentum D) Conservation of linearity

Answer: B.

12. **The operator equation of motion in Schrödinger picture is expressed by:**A) Heisenberg equation of motion B) Newton's second law C) Probability conservation D) Dirac equation

Answer: A.

13.In an infinite square well (0 \leq x \leq L), ψ is: A) Sin(kx)+cos(kx) B) Zero outside 0–L C) An exponential inside D) Constant inside

Answer: B

14.**Allowed wavefunctions in infinite well must satisfy $\psi(0) = \psi(L) = 0$ because:** A) Operator Hermiticity B) Continuity & infinite barrier C) Probability normalization D) Discreteness of energy

Answer: B

15. Energy levels in infinite well are proportional to n^2 because: A) Length quantization B) $k = n\pi/L \rightarrow E \propto k^2$ C) Angular momentum rules D) Potential is zero

Answer: B

16. In the ground state of infinite well, (x) =: A) 0 B) L/2 C) L D) Undefined

Answer: B

17.**Wavefunction for finite square well outside the well decays:** A) As sine B) As cosine C) Exponentially D) Oscillatory

Answer: C

18.**A finite well allows which phenomenon not present in infinite well?** A)
Zero energy ground state B) Tunneling C) No bound states D) Infinite
discrete levels

Answer: B

19.**In finite well bound states require $E < V_0$ so solutions are:** A) Oscillatory everywhere B) Oscillatory inside, exponential outside C) Exponential everywhere D) Oscillatory outside only

Answer: B

20. The minimum non-zero energy in infinite well follows from uncertainty because E= $p^2/2m$ and $p \approx \hbar/\Delta x$. This forbids E = 0. A) True B) False

Answer: A

21. **Rectangular barrier above particle energy leads to:** A) Full reflection only B) Tunneling with finite probability C) No quantum effect D) Infinite penetration

Answer: B.

22. **Potential step with E > V step gives:** A) Only reflection B) Only transmission C) Partial reflection and transmission D) Zero chance of crossing

Answer: C.

23.**1D Harmonic oscillator eigenfunctions are**: A) Sines & cosines B) Hermite polynomials × Gaussian C) Laguerre polynomials D) Legendre functions

Answer: B

24. Energy eigenvalues for 1D oscillator: A) En \propto n² B) En = ħω(n + ½) C) En = nħω D) En continuous

Answer: B.

25. Particle in a 3D box has energies separable as: A) Sum of 1D energies in x, y, z B) Product of sine functions only C) Single-n quantum number D) Degenerate in all states

Answer: A.

26. **Radial part of hydrogen atom wavefunction uses:** A) Hermite polynomials B) Associated Legendre functions C) Associated Laguerre polynomials D) Bessel functions

Answer: C

27.3D hydrogen radial probability distribution peaks at bohr radius a₀ for n = 1: A) True B) False

Answer: A.

28. Which equation defines the radial probability? A) $|\psi(r)|^2$ B) $|R(r)|^2r^2$ C) $|Y(\theta,\varphi)|^2$ D) $|\psi(r)|^2r$

Answer: B.

29. Solving associated Laguerre differential equation yields hydrogen bound states A) True B) False

Answer: A.

30. Non-commutativity of operators is unrelated to energy quantization: A) True B) False

Answer: B – it's crucial for deriving uncertainty which underpins quantization.